



Single machine scheduling in a batch delivery system with fixed delivery dates

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ABSTRACT

This paper presents a new mathematical model for scheduling of a single machine in a batch delivery system. The objective is to minimize sum of the delivery and total holding costs. Accordingly, the proposed model tries to find a solution with high quality (good solution) by a tradeoff between the holding and delivery costs. Two examples are solved by the LINGO Software to validate and verify the proposed model. The mathematical model attempted in this paper suffers from a large number of variables and constraints which make application of this model difficult for large-sized real cases. Thus meta-heuristic approaches can be developed and applied for solving the real-sized problems in future researches.

Original Article

PII: S232247701800003-8

Rec. 30 Jan. 2018
Acc. 28 Feb. 2018
Pub. 25 Mar. 2018

Keywords

Single machine scheduling;
Fixed delivery date;
Batch delivery system

INTRODUCTION

A standard assumption in classical scheduling is that a job is delivered to its customer immediately after its completion time. However, there are situations in which jobs are delivered on a set of predetermined fixed delivery dates, which are externally given or internally determined by taking advantage of economies of scale in delivery cost, downstream parties, or periodic business considerations, and so on.

Matsuo (1988) was probably the first researcher who considers machine scheduling with fixed delivery dates. Considering some scheduling problems with fixed delivery dates, Lesaoana (1991) analyzed their computational complexity and provides solution algorithms. Hall et al. (2001) considered various scheduling problems in which the number of fixed delivery dates is either constant or specified as part of the data input. They analyze the computational complexity of the problems and

provide some pseudo polynomial time algorithms. Xiuli and Cheng (2011) proposed largest ratio first (LRF) rule that sequences jobs in non-increasing order of w_j/p_j , where p_j and w_j are the processing time and weight of job j respectively, for minimization total weighted flow time. In this paper, is considered the single-machine scheduling problem with fixed delivery dates which are given before the jobs are processed. The holding time is defined as duration that job completed but not yet delivered its. The objective is to minimize the total holding costs of the jobs and delivery costs of the batches. The recognition version of this problem is NP-hard (Hall et al., 2001).

The remainder of the paper is organized as follow: Section 2 represents a formal description of the problem and mathematical model. Computational analysis is presented in Section 3. Finally; Section 4 presents the general conclusion and suggestion for future research.

METHODOLOGY

Problem description & mathematical model

In this study, a single machine scheduling problem with fixed delivery dates is considered, delivery dates are predefined. The objective is to minimize the total job's holding cost and delivery cost of batch, meaning of holding time is duration that job is completed but not yet delivered. In this paper endeavors to determine which fixed delivery dates selected to dispatch batch and sequence of the jobs in batch its (Figure 1).

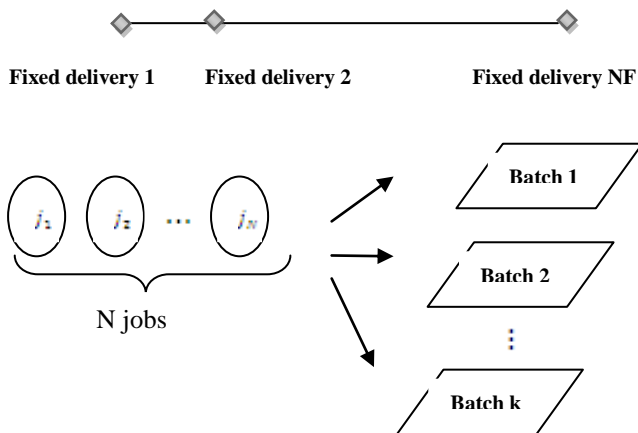


Figure 1. structure of batch delivery system with fixed delivery dates

In this problem some assumption are presumed that are listed in the following:

- The jobs are available (release date of jobs are zero)
- There is no precedence constraint between jobs.
- Processing of job must be completed and can't interrupted (preemption isn't allowed).
- Fixed delivery dates are predefined.
- Processing time and holding cost of jobs are predefined.

Indexes

$j = 1, 2, 3, \dots, N$ Index of the Sequence jobs on the machine

$i = 1, 2, 3, \dots, N$ Index of the jobs

$k = 1, 2, 3, \dots, NF$ Index of the fixed delivery dates

Parameters

P_i =Processing time of the job i

h_i =Holding cost of the job i in the per unit of time

D_k =Fixed delivery date k

δ = delivery cost

Decision variables

$x_{ijk} = \begin{cases} 1 & \text{if job } j \text{ in the position } i \text{ delivered at } k \text{ th fix delivery date} \\ 0 & \text{otherwise} \end{cases}$

$q_k = \begin{cases} 1 & \text{if a batch is dispatched in the } k \text{ th fix delivery date} \\ 0 & \text{otherwise} \end{cases}$

C_i = completion time of the job i

W_i = holding time of the job i

Mathematical modeling

The problem is formulated as follows:

$$\text{Min } z = \sum_{i=1}^N w_i \cdot h_i + \delta \cdot \sum_{k=1}^{NF} q_k \quad (1)$$

St:

$$\sum_{i=1}^N \sum_{k=1}^{NF} x_{ijk} = 1 \quad \forall j \quad (2)$$

$$\sum_{j=1}^N \sum_{k=1}^{NF} x_{ijk} = 1 \quad \forall i \quad (3)$$

$$x_{ijk} \leq q_k \quad \forall i, j, k \quad (4)$$

$$1 \leq \sum_{k=1}^{NF} q_k \leq \text{Min}\{N, NF\} \quad (5)$$

$$C_i = \sum_{j=1}^N \sum_{k=1}^{NF} x_{ijk} \cdot (\sum_{j' \leq j} \sum_{i'} \sum_{k'} x_{i'j'k'} \cdot p_{j'}) \quad \forall i \quad (6)$$

$$w_i = \sum_{j=1}^N \sum_{k=1}^{NF} x_{ijk} \cdot (D_k - C_i) \quad \forall i \quad (7)$$

$$x_{ijk}, q_k = \{0, 1\}, w_i, C_i \geq 0 \quad (8)$$

The objective function, minimizing the sum of the total holding costs and delivery costs is expressed by eq. (1). eq. (2) ensures that in the position of sequence only one job placed. eq. (3) ensures that each job is assigned only to one position in the sequence of jobs and that's delivered only to one fixed delivery dates. eq. (4) Represents that each fixed delivery date is opened if at least one job is delivered in this fixed delivery date .eq. (5) ensures that minimum and maximum of batches in the batch delivery system are equal to 1 and $\text{Min}\{N, NF\}$ respectively. eq. (6) indicates the completion time for each job .eq. (7) calculates holding time for each job, the holding time of job j is as duration that it's completed but not delivered. eq. (8) indicates decision variables.

The number of variables and constraints in the model are presented in Tables 1 and 2, respectively, based on the variable indices.

Table 1. The number of variables in the model

Variable name	Variable count
x_{ijk}	N.N.NF
q_k	NF
w_i	N
C_i	N

Table 2. The number of constraints in the model

Equation number	Constraint count
1	N
2	N
3	N.N.NF
4	1
5	N
6	N

RESULTS AND DISCUSSION

Computational analysis

In order to provide a better understanding and verify the behavior of the proposed model, two numerical examples are presented to show applicability of the model. These examples are defined by the decision maker. These examples are solved by branch and bound (B & B) method with Lingo 9 software package.

Example 1.

This example includes two fixed delivery dates and four jobs. The data set related to the processing time and holding cost of each job are shown in tables 3 and 4. Delivery cost (δ) is equal to 100. For example, processing time for job1 is 5 and holding cost for this job is 10. Table 5 shows the results obtained by the proposed model. For this example objective function is equal to 525.

Table 3. Job-processing time and holding cost of example1

Number of job	1	2	3	4
Processing time	5	10	1	20
Holding cost	10	5	20	2

Table 4. Fixed delivery dates of example1

Fixed delivery dates	1	2
	25	50

Table 5. Results of example1

Number of job	1	2	3	4
Completion time	31	36	21	20
Holding time	14	19	4	5

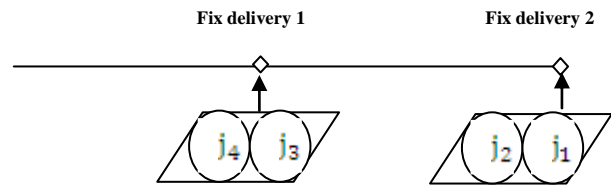


Figure 2. Graphical results of example1

Example 2.

This example includes three fixed delivery dates and five jobs. Tables 6 and 7 show the related data for example 2 with delivery cost (δ) is equal to 100. Table 8 indicates the results obtained by the proposed model for this example. The objective function value is 435.

Table 6. Job-processing time and holding cost of example 2

Number of job	1	2	3	4	5
Processing time	5	10	10	20	7
Holding cost	10	5	20	2	5

Table 7. Fixed delivery dates of example 2

Fixed delivery dates	1	2	3
	25	40	60

Table 8. Results of example 2

Number of job	1	2	3	4	5
Completion time	52	30	40	20	47
Holding time	8	10	0	20	13

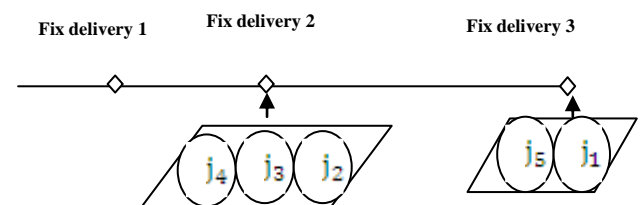


Figure 3. Graphical results of example2

■ CONCLUSION

In this paper, a new mathematical model is proposed for batch delivery system with fixed delivery dates. The objective is to minimize total holding and delivery costs. The mathematical model attempted in this paper suffers from a large number of variables and constraints which make application of this model difficult for large-sized real cases. Thus meta-heuristic approaches can be developed and applied for solving the real-sized problems in future researches.

■ DECLARATIONS

Authors' Contributions

All authors contributed equally to this work.

Competing interests

The authors declare that they have no competing interests.

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